What is light?

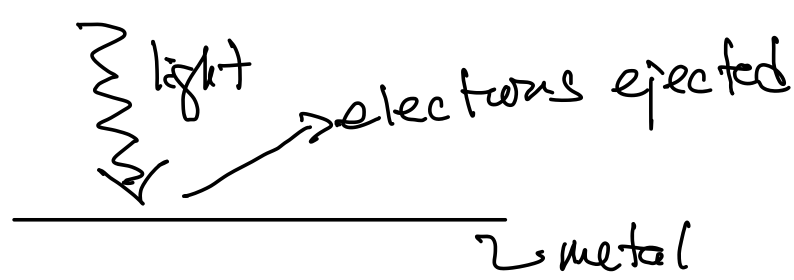
- Newton, early 1700s: particles
  geometric optics, reflection, refraction
- Huygens, 1700 + Young, early 1800s
  interference, diffraction, wavelets,
  superposition

by mid 1850s, wave theory was dominating over particle theory

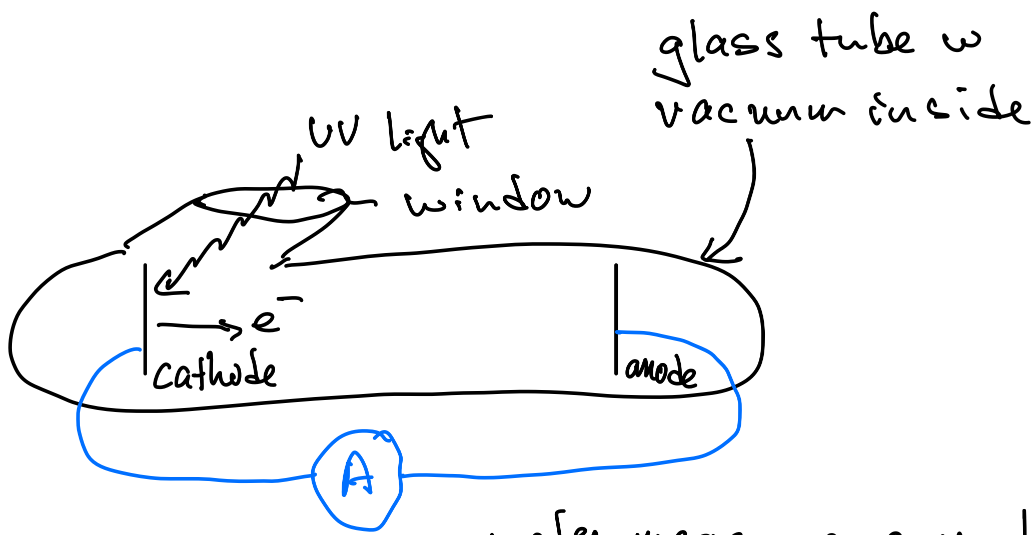
Photoelectric effect

1839, Becquerel discovered photovoltaic effect
→ light exposure on some materials generates voltages

late 1800s, experiments with metals:



Phillip Lenard experiment, 1900



ammeter measures current flow

⇒ shine UV light onto cathode, registers a current in the ammeter

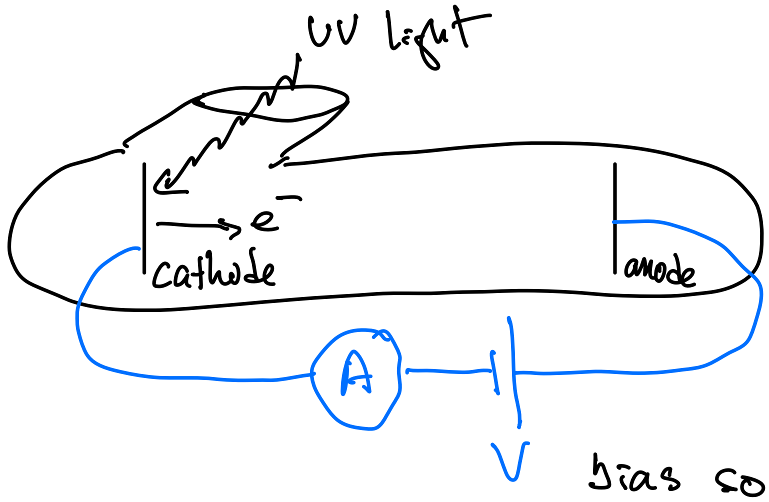
saw that: no light, no current

⇒ current in space between electrodes must be same as current in the wires thru meter

can measure intensity of current I_c vs intensity of light I_ℓ

$$I_c \propto I_\ell$$

now add battery to circuit



bias so anode is at $+V$ relative to cathode

$\Rightarrow$ this will accelerate electrons to anode

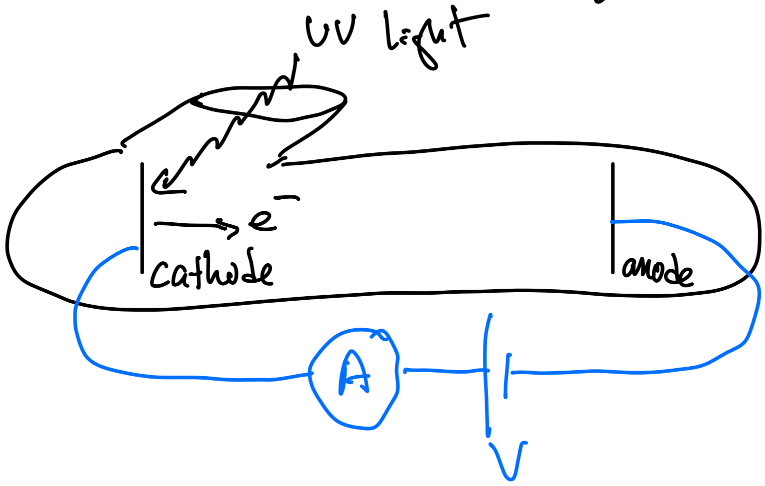
Energy gained by electrons: $\Delta E = eV$

However, Lenard observed that the current thru the ammeter did <u>not</u> change with increasing voltage

but still observed $I_e \propto I_\ell$

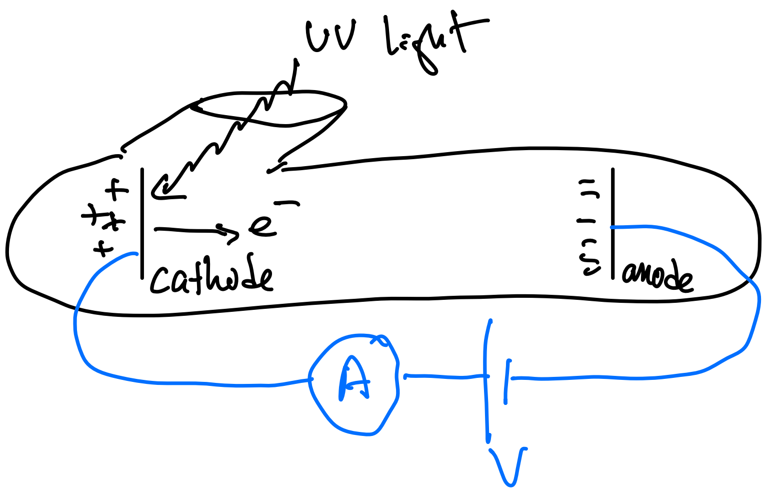
$\Rightarrow$ more light, more current

next reverse polarity of battery



Here, elections would have to overcome anode repulsion

⇒ Because battery is pumping + charges to cathode!



elections will need $KE \geq eV$ to get to anode

⇒ Lenard observed that as V increases, current dropped but can make up for it by increasing light current

However: <u>only</u> up to some voltage V_s (stopping voltage)

and, no matter how much you crank up light intensity I_ℓ, still no current as long as $V > V_s$!

$\Rightarrow$ This implies that KE of ejected electrons is <u>not</u> proportional to light intensity!

Next Lenard increase light frequency and found current I_e turned on!

so $I_c \propto f_{light}$ when $V > V_s$

this implies that KE of electrons ejected is proportional to the light frequency and <u>not</u> light intensity

Problems w/ "classical" picture of photo-electric effect

1. at some stopping voltage, current $\to 0$
   expect current to reappear by increasing light intensity
   result: • current intensity does not depend on light intensity!
   • current reappears by raising light frequency!

2. it takes some energy ("work function", ϕ) to free an electron from the metal surface

   $\Rightarrow$ any electron would have to absorb energy from light intensity to build up enough to get free

   $\Rightarrow$ therefore there should be a delay between seeing a current & turning on light

   result: no delay seen – current intensity appears immediately!

Resolution: Einstein, 1905

1. Photons are particles
2. Energy of photon $\propto$ photon frequency:

proposed by Planck in ~1900

$$E = hf \qquad h \equiv \text{Planck's constant}$$

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

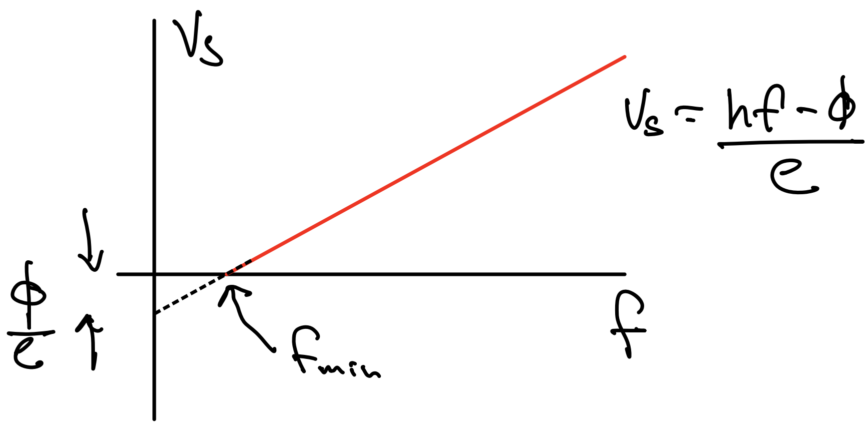
3. each electron absorbs single photon and gains energy $E = hf$

to get to anode w/ stopping voltage V_s and work function ϕ:

$$KE = hf - \phi \geq eV_s$$

experiment to do

1. shine light w/ some frequency f
2. find V_s that causes current to vanish
3. vary f, measure V_s, plot:



- Positive slope gives $\frac{h}{e}$
- y-intercept gives $V_s(f=0) = -\phi/e$

Verified! Theory fit the data!

note: for any particle w/ charge q going thru an electric potential ΔV, it gains energy:

$$\Delta E = q\Delta V$$

if $q = e \equiv 1.6\times10^{-19}C$ and ΔV is measured in volts then the energy $\Delta E = e\Delta V$

ex: $e = 1.6\times10^{-19}C$ charge of proton & |electron| and $\Delta V = 1$ volt:

$$\Delta E = 1.6\times10^{-19}C \cdot 1V = 1.6\times10^{-19} J \text{ (joules)}$$

we define "electron-volt" eV as

$$\boxed{1eV = 1.6\times10^{-19}C}$$

unit of energy gained by electron thru 1 volt

Work functions

Depends on metals:

Alum.	4.3eV
Carbon	5.0eV
Copper	4.7eV
Gold	5.1eV
Nickel	5.1eV
Silicon	4.8eV
Silver	4.3eV
Sodium	2.7eV

Very small!

But remember $E = hf$ for photon

ex: light w/ $\lambda = 400\,nm$

$c = \lambda f$ so $f = c/\lambda$

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,J\text{-}s * 3\times10^{8}\,m/s}{400\times10^{-9}\,m}$$

$$= 5.0\times10^{-19}\,J \quad \text{small!}$$

then in eV, $E = 5\times10^{-19}\,J * \frac{1eV}{1.6\times10^{-19}\,J} = 3.1eV$

so 400nm is not going to eject electrons except in sodium

⇒ that's why UV (ultraviolet) is used

note: $hc = 6.63\times10^{-34}\,J\text{-}s \times 3\times10^{8}\,m/s$

$= 1.989\times10^{-25}\,J\text{-}m \times \frac{1\,eV}{1.6\times10^{-19}\,J}$

$= 1.243\times10^{-6}\,eV\text{-}m$

$= 1.243\,eV\text{-}\mu m$

$= 1243\,eV\text{-}nm$

This lets you convert from wavelength to energy!

ex: $\lambda = 400\,nm$

$E = \frac{1243\,eV\text{-}nm}{400\,nm} = 3.1\,eV$

ex: laser pointer w/output 5mW emits red light, $\lambda = 650\,nm$

energy of each photon $E = \frac{hc}{\lambda} = \frac{1243\,eV\text{-}nm}{650\,nm}$

$= 1.9\,eV$

in joules: $1.9\,eV \times \frac{1.6\times10^{-19}\,J}{eV} = 3.1\times10^{-19}\,J$

each electron will have $KE = \frac{1}{2}mv^2$

- elections that are inside close to surface will need to overcome work function ϕ to get out
- elections already on surface will absorb entire photon energy

$\Rightarrow$ so max $KE = hf = 1.9\,eV$ elections on surface

$$KE = \frac{1}{2} m_e v^2 = 1.9\,eV = 3.1\times10^{-19}\,J$$

$$v = \sqrt{\frac{2 * KE}{m_e}}$$

$$m_e = 9.1\times10^{-31}\,kg$$

$$v = \sqrt{\frac{2\times3.1\times10^{-19}\,J}{m_e}} = \sqrt{\frac{2\times3.1\times10^{-19}\,J}{9.1\times10^{-31}\,kg}}$$

$$= 8.25\times10^{5}\,m/s$$

write: $m_e v^2 = m_e c^2 \left(\frac{v}{c}\right)^2 = m_e c^2 \beta^2$

$m_e c^2$ = rest mass of election

$$= 9.1\times10^{-31} * (3\times10^{8})^2 = 8.2\times10^{-14}\,J$$

in eV: $m_e c^2 = 8.2\times10^{-14}\,J * \frac{1\,eV}{1.6\times10^{-19}\,J} = 511\times10^{3}\,eV$

$= 511\,keV$

then $\frac{1}{2} m_e v^2 = \frac{1}{2}(m_e c^2)\beta^2 = 1.9\,eV$

$$\beta = \sqrt{\frac{2 * 1.9\,eV}{511 \times 10^3\,eV}} = 2.7 \times 10^{-3}$$

$$v = \beta c = 2.7 \times 10^{-3} * 3 \times 10^8 \frac{m}{s} = 8.2 \times 10^5 \frac{m}{s}$$

<u>Photon energy & momentum.</u>

$$E = hf \quad \text{and} \quad c = f\lambda \quad \text{so} \quad f = c/\lambda$$

so $E = \frac{hc}{\lambda}$

Special relativity: $E^2 = (pc)^2 + (m_0 c^2)^2$

from other experiments: $m_0 = 0$ for photons (γs)

so $E = pc$

$$p = E/c = \frac{hc}{\lambda}/c = \frac{h}{\lambda}$$

$$\boxed{\left.\begin{aligned} E &= \frac{hc}{\lambda} \\ p &= \frac{h}{\lambda} \end{aligned}\right\} E = pc}$$ for photons

Light intensity I in photon picture

$I = \eta c$ for EM waves

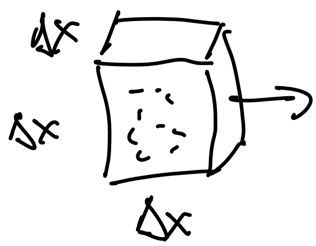
η = energy density = $\frac{1}{2}\varepsilon_0 E^2$ for EM waves

intensity I = flux, units of $\frac{\text{Power}}{\text{area}} = \frac{\text{Energy/sec}}{\text{area}}$

is the rate of energy flow per time thru an area

in photon picture:

take a volume $\Delta V = \Delta x^3$ full of photons flowing thru



thru area $\delta A = \Delta x^2$

photons per volume $\equiv n$

these go distance $\Delta x = c\Delta t$ in time Δt

$r \equiv$ photon flux $\equiv$ # photons/area/time

$$\frac{\#\text{photons}}{\text{area}\cdot\Delta t} = \frac{\#\text{photons}}{\text{volume}} * \frac{\Delta x}{\Delta t} = nc$$

if each photon has energy $E = hf$ then energy density is $E\cdot n \Rightarrow \eta = hf\cdot n$

so $r = nc = \frac{\eta}{hf} \cdot c = \frac{I}{hf}$

so $\boxed{I = hf \cdot r}$ $r =$ # γ per area per sec

$I = \frac{\text{power}}{\text{area}} = \frac{\text{energy}}{\text{area} \cdot \text{sec}} = \frac{\text{energy}}{\text{photon}} \times \frac{\text{photons}}{\text{area} \cdot \text{sec}}$

so $I = hf \cdot r$

this relates wave intensity to photon flux

where flux $= \frac{\#}{\text{area} \cdot \text{sec}}$

can define $R =$ # photons/sec hitting surface of area A

$\therefore \quad = r \cdot A$

and $I = \frac{P}{A}$ power per area

so $I = \frac{P}{A} = hf \cdot r = hf \cdot \frac{R}{A}$

or $\boxed{P = hf \cdot R}$ $R =$ # γ per sec ex: laser pointer has 5mW power and $\lambda = 650\,nm$

$$R = \frac{P}{hf} = \frac{P}{hc/\lambda}$$

$$= \frac{5\times10^{-3}\,J/s}{1243\,eV/nm} * 650\,nm * \frac{1\,eV}{1.6\times10^{-19}\,J}$$

$$= 1.6\times10^{16}\ \gamma/sec$$

ex: sun intensity is $\sim 1400\,W/m^2$ at top of atmosphere

if all those γ's were $\sim 400\,nm$, how many photons hit an area of $1\,m^2$ per sec?

$$1400\,\frac{W}{m^2} * 1\,m^2 = 1400\,W = 1400\,\frac{J}{S}$$

total energy in $J = \#\gamma\text{'s} * \frac{\text{energy}}{\text{photon}} = N_\gamma \cdot hf$

$$= N_\gamma \frac{hc}{\lambda} = N_\gamma \cdot \frac{1243\,eV\text{-}nm}{400\,nm}$$

$$J = N_\gamma \cdot 3.1\,eV$$

$$1400\,\frac{J}{S} = \frac{N_\gamma \cdot 3.1\,eV}{S} \quad \text{so} \quad \frac{N_\gamma}{S} = \frac{1400\,J}{3.1\,eV} * \frac{1\,eV}{1.6\times10^{-19}\,J}$$

$$= 2.8\times10^{21}\ \gamma/S$$

lots of γ's per sec!

X-ray production

Produce a beam of electrons:

1. heat up a coil, usually tungsten

tungsten melting temp 3400°C = 6200°F

heat energy absorbed by electrons

- energy equivalent of heat is via Boltzmann law:

$$Q(\text{heat energy}) = k_B T$$

$$k_B = 8.6 \times 10^{-5}\ eV/K = 1.38 \times 10^{-23}\ J/K$$

room temp ~ 300K

so $Q(\text{room temp}) = 8.6 \times 10^{-5} \frac{eV}{K} * 300K$

$= 0.026eV \sim \frac{1}{40} eV$

energy of 3400°C : $E = k_B T = 8.6 \times 10^{-5} \frac{eV}{K} * 3400K$

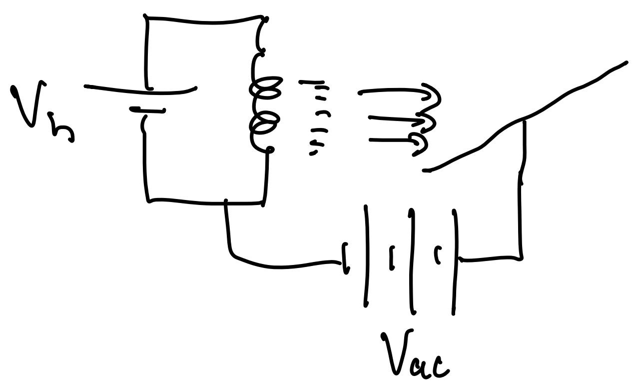
$= 0.29eV$

this is below work function value for tungsten ~ 4.5eV

But this is just the average - many

electrons will absorb more heat and have enough to escape tungsten surface

2. accelerate electrons using positive voltage



what happens when electrons hit plate?

Wave picture:

- plate heats up & emits photons (same effect as moving electrons from tungsten)
- spectrum of waves emitted -> continuous (no specific energies)

Photon picture:

- electrons all have ~ same energy eV_{ac}
- γ's released will also have same energy from electrons losing all of its energy stopping

experiment confirms photon picture

ex: elections accelerated thru $V_{ac} = 50V$

$E = 50eV$ = energy given to photons

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{1243 eV\text{-}nm}{\lambda} = 50eV$$

$$\lambda = \frac{1243}{50} = 24\,nm$$

λ for x-rays is $\sim 0.01 \to 10\,nm$

Dental x-rays use $V_{ac} \sim 60\text{-}70\,kV$

$$E = eV_{ac} = \frac{1243 eV\text{-}nm}{\lambda}$$

for $V_{ac} = 60\,kV$, $E = 60\,keV = 60 \times 10^3\,eV$

$$\lambda = \frac{1243\,eV\text{-}nm}{60 \times 10^3\,eV} = 0.02\,nm$$

Waves & Particles?

Light diffracts & interferes ⇒ waves

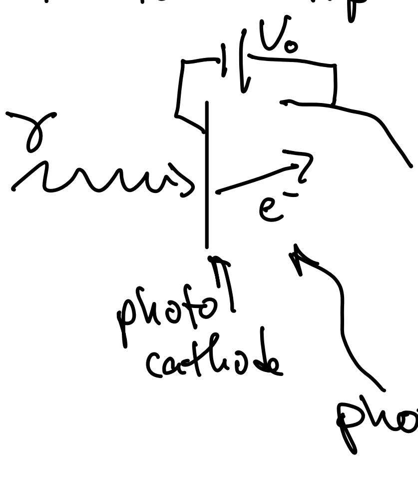
Light consists of photons ⇒ particles?

Which is correct? <u>Both</u>

If you do an experiment to measure wave-like properties, it will behave like a wave.

Same for particle-like properties

Photo-multiplier tube: detects photons:

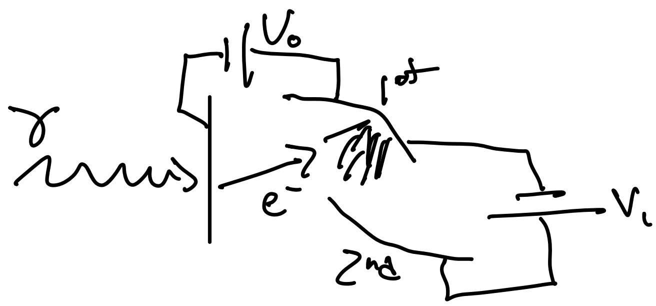


electron is accelerated to 1st stage, which is at a $+V_0$ potential

e^- gains energy eV_0 and hits plate

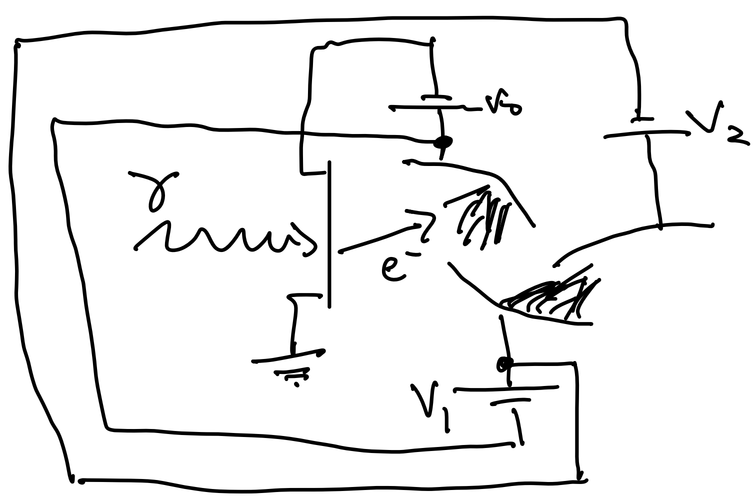
this kicks off more electrons that share eV_0 energy

⇒ next place 2nd stage w/ potential V_1 above 1st accelerate electrons from 1st stage to 2nd stage



secondary electrons each gain energy eV_1 and hit 2nd stage

2nd stage produces tertiary electrons that head to 3rd stage that is at V_2 above 2nd stage



ect. Each stage produses N x previous stage.

if you have n stages, final current of electrons:

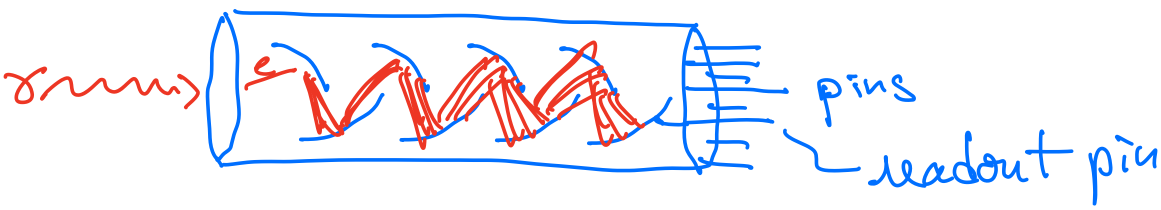
$$I = \frac{\Delta Q}{\Delta t} = \frac{N^n \cdot e}{\Delta t}$$

these are fast, $\Delta t \approx 10\,ns$

if you start w/ 1 photon and have $N=10$ produced at each stage in 10 ns:

$$I = \frac{10^{10} \cdot 1.6\times10^{-19}\,C}{10\times10^{-9}\,s} = 0.16 \text{ Amps}$$

so it's a photo-multiplier



- Built into cylindrical form factor
- pins at end are for various stage voltages and final stage readout pin

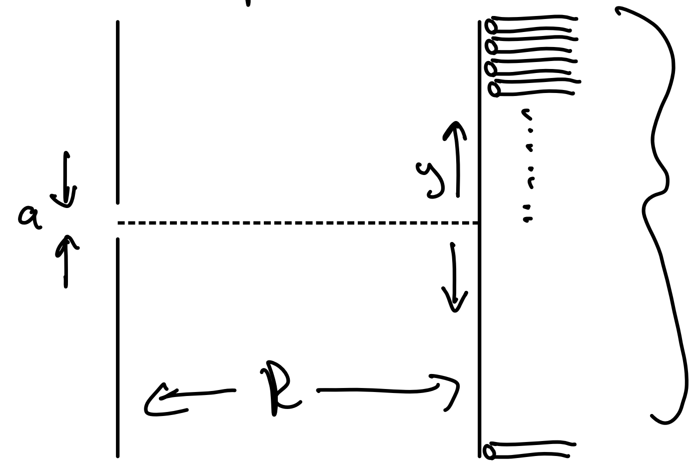
that's why it's called a photo-multiplier tube (PMT)

can be used to detect individual photons

Diffraction: wave picture gives

$$I = I_0 \operatorname{sinc}\left(\frac{\pi a \sin\theta}{\lambda}\right)$$ a = slit width
single slit

in Particle picture:



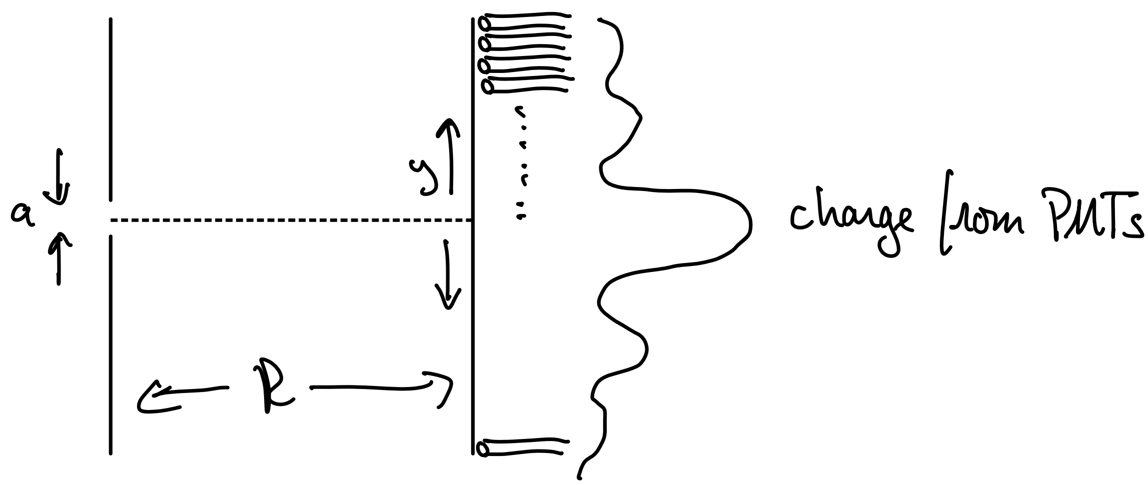
cover screen with PMTs

- Readout each tube every second & integrate current (charge)
- make plot of # times PMT had a "hit", vs y coord

will get same $\operatorname{sinc}\left(\frac{\pi a \sin\theta}{\lambda}\right)$ distribution

where as before $\sin\theta \sim \theta = y/R$

Resulting distribution of PMT hits:



Both ways of measurement gives same result:

1. waves → measure intensity

2. particles → measure PMT hit spatial distribution

Both give equivalent results
But waves are very different from particles!

- light has wave-like characteristics
- light is made up of particles (photons)

So what is the connection between particles & waves?

Correspondence between:

- ⇒ wave intensity I (power per area per sec)
- ⇒ photon flux Γ (photons per area per sec)

is $I = \Gamma \cdot hf$

If you turn down the light intensity, you also reduce # photon emitted.

Keep decreasing intensity.
Eventually you get down to a single γ and you can't reduce intensity further & still have light.

So... we say that light is "quantized"

⇒ any light intensity consists of some <u>huge</u> # of photons

This is a basis for "quantum mechanics"

But... even w/ light of only 1 photon, it's still a wave.

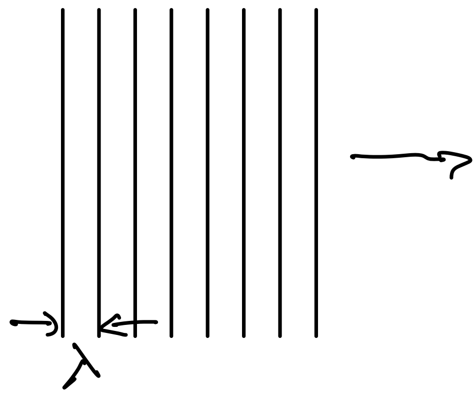
⇒ photon energy $E = hf$ and momentum $p = \frac{h}{\lambda}$

⇒ what is the wave nature?

Waves & particles

waves have wavelength $k = \frac{2\pi}{\lambda}$

wavefront :



has definite wavelength

but is not localized → wave exists along infinite wavefront

particles have position coordinates

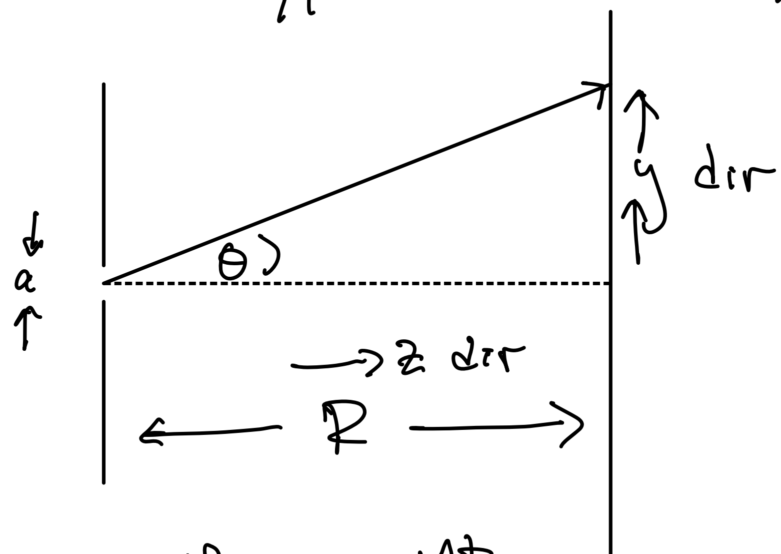
⇒ but a particle not moving does not have a definite wavelength

⇒ this implies that in wave-particle picture there is a relationship between the degree that you can know position: uncertainty in $x \Rightarrow \Delta x$

momentum: " " $p \Rightarrow \Delta p$

(thus $p = h/\lambda$ relation)

Single slit diffraction & wave/particle



from wave theory: 1^{st} minimum is at

$$a \sin\theta_1 = \lambda$$

uncertainty in wave position in slit:

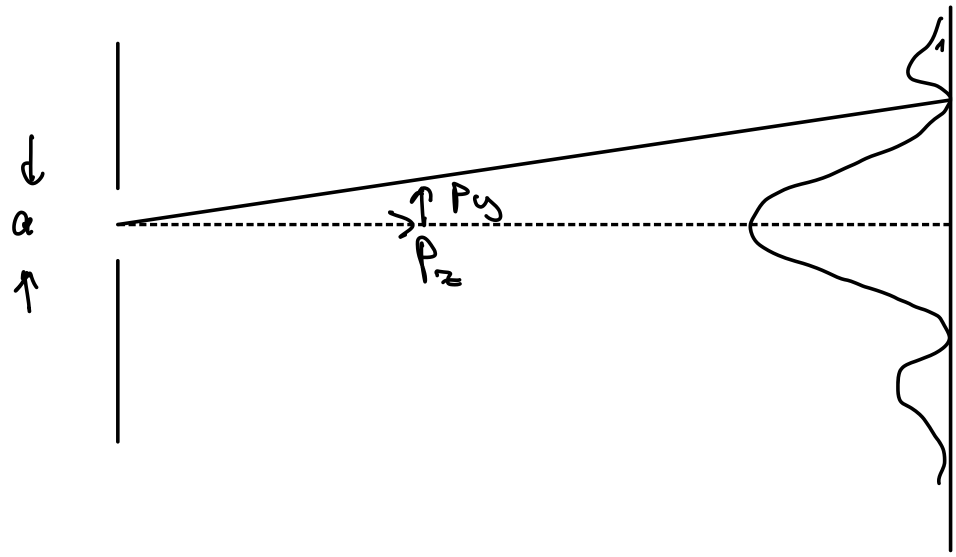
$$\Delta y = a$$

wave has a momentum along z direction:

$$p_z = \frac{h}{\lambda} = \hbar k$$

for photon to get to 1^{st} min angle, it would have to pick up some unknown momentum Δp_y

and can write $\sin\theta \sim \tan\theta = \dfrac{\Delta p_y}{p_z}$



so $\quad a\sin\theta \to \Delta y \cdot \dfrac{\Delta p_y}{p_z} = \lambda$

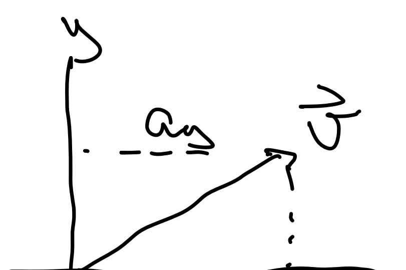
or $\quad \Delta y\, \Delta p_y \sim \lambda p_z = \lambda \dfrac{h}{\lambda} = h$

$$\Delta y\, \Delta p_y \sim h$$

this kind of relationship is common in classical wave mechanics

Fourier analysis: extension of vector mathematics on to functions

vectors



$\vec{v} = a_x \hat{i} + a_y \hat{j}$

$\hat{i}$ points along x

$\underbrace{\hat{j} \quad " \quad " \quad y}_{\text{"basis vectors"}}$

rules: 1. $\hat{i} \perp \hat{j}$ so $\hat{i} \cdot \hat{j} = 0$

2. $\hat{i} \cdot \hat{i} = 1$ and $\hat{j} \cdot \hat{j} = 1$

how to calculate a_x & a_y:

$$\left.\begin{aligned} a_x &= \vec{v} \cdot \hat{i} \\ a_y &= \vec{v} \cdot \hat{j} \end{aligned}\right\} \text{"inner products"}$$

so we write $\vec{v} = \sum_{i=1}^{2} a_i \hat{k}_i$

where $\hat{k}_1 = \hat{i}$ & $\hat{k}_2 = \hat{j}$

and $a_i = \vec{v} \cdot \hat{k}_i$

Fourier theorem:

can take any periodic function $f(\theta)$ and expand just like a vector:

$$f(\theta) = \sum_{i=1}^{\infty} a_i g_i(\theta) \leftarrow \text{basis functions}$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$\vec{v} = \sum_{i=1}^{2} a_i \hat{k}_i$$

calculate a_i same way:

$$a_i = \int_{-\infty}^{\infty} f(\theta) g_i(\theta) \, d\theta$$

for periodic functions $f(\theta)$, can use trig for the basis functions

$$g_n(\theta) = \sin n\theta \quad \text{or} \quad \cos n\theta$$

this works because of the condition:

$$\hat{i}\cdot\hat{j} = 0 \Rightarrow \int_0^{2\pi} g_n(\theta)\, g_m(\theta)\, d\theta = 0 \quad \text{unless } n=m$$

ex: $g_n(\theta) = A\cos n\theta$

$$\int_0^{2\pi} A\cos n\theta \cdot A\cos m\theta\, d\theta$$

each function spends half of θ above & below so integral $= 0$

<u>unless</u> $n=m$:

$$\int_0^{2\pi} A^2\cos^2 n\theta\, d\theta = A^2 \int_0^{2\pi} \tfrac{1}{2}(1+\cos 2n\theta)\, d\theta$$

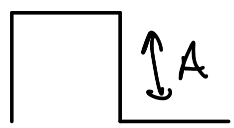
$$= \frac{A^2}{2}\cdot 2\pi = \pi A^2 \neq 0\,!$$

set $A^2 = \frac{1}{\pi}$ to get $\int g_n^2(\theta)\, d\theta = 1$

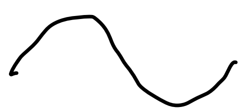
Fourier expansion: $f(\theta)=\sum_{i=1}^{\infty} a_i g_i(\theta)$

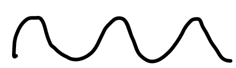
but of course we might not need to let $i \to \infty$ if it converges fast enough

ex: square wave

square wave: 

can be constructed from:

= 

\+ 

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the Fourier expansion for square wave:

$$f(x) = \frac{A}{2}\left(1 + \frac{4}{\pi} \sum_{n=1,3,5\ldots} \frac{\sin(nkx)}{n}\right.$$

you decide on how many terms in n to keep to approximate $f(x)$

But

the more localized the wave, the more wavelengths you have to add to describe it accurately so there will be a "spread" (uncertainty) in the wavelengths added:

$k = \frac{2\pi}{\lambda}$ wave number, = # wavelengths per meter

$\Rightarrow$ As you localize wave more, the position uncertainty decreases $\Delta x \to 0$ (x along slit)
for wave w/ wavefront, $\Delta x \to \infty$ along the wavefront that extends to ∞

so infinite Δx has definite $k = \frac{2\pi}{\lambda}$
and very small Δx (localized) needs more and more waves superimposed, each w/ different frequencies (Fourier)

so there's an uncertainty relation (can be proved)

$$\Delta x \, \Delta k \geq \frac{1}{2}$$

This is purely wave mechanics, from studying Fourier analysis
But it applies to QM:

$p = \frac{h}{\lambda}$ and $k = \frac{2\pi}{\lambda}$ so $\frac{1}{\lambda} = \frac{k}{2\pi}$

so $p = \frac{h}{2\pi} k$ write $\frac{h}{2\pi} \equiv \hbar$ (h-bar)

$hc = 1243$ eV-nm

so $\hbar c = \frac{hc}{2\pi} = 197.8$ eV-nm

Apply Fourier wave mechanics to QM:

$\Delta k = \frac{\Delta p}{\hbar}$ and $\Delta x \Delta k \geq \frac{1}{2}$

so $\boxed{\Delta x \Delta p \geq \frac{\hbar}{2}}$ Heisenberg uncertainty formula

This is a deep & fundamental law of nature

Heisenberg $\Delta x \, \Delta p_x \geq \hbar/2$

Very fundamental to QM:

$\Rightarrow$ you can not (in principle) know precisely both position and momentum

also there's an uncertainty relation for energy:

$$\Delta E \, \Delta t \geq \hbar/2$$

$\Rightarrow$ cannot localize in time and know energy precisely

The more localized in position, the less definite the momentum

& Vice versa

But the theory also says more:

- The position & momentum do not exist until measured!
- only the probability for having some value of position and momentum exist
- The wave equation tells you this probability

It is not that you don't know position & momentum

=> its that a definite position & a definite momentum do not exist

reality at subatomic (quantum) level:

the only thing that exists is probability

=> This might seem odd or wrong but all technology is based on it

Schrodinger's Cat

put cat in box w/ radioactive source

QM says:

1. cannot predict when source will decay
2. can predict probability as function of time that has decayed
3. can make many measurements, construct probability, compare w/ theory - agrees

4. before measuring, decay is not a valid concept → particle is in a superposition of "decayed" and "not decayed"

⇒ state of decayed or not <u>only</u> is real once you make the measurement

⇒ this is called "collapse of wave function"

Schrodinger cat → we still don't agree about it!